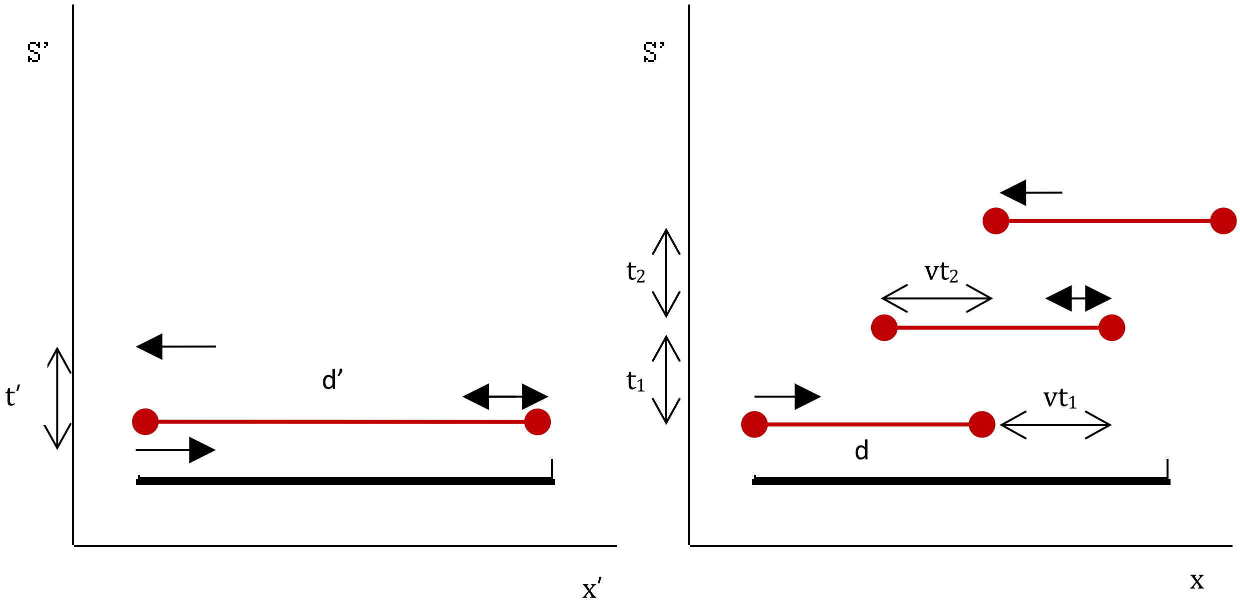
Activity 2 - Length Contraction

We have seen that time intervals between events appear to be different lengths depending upon the relative speeds of observers in different inertial frames. Special relativity gets even stranger still. If someone is moving relative to us they actually look thinner in the direction of travel!



Scientists in a spaceship, frame S', decide to measure a rod, which is stationary in S', by placing mirrors on its ends and timing how long it takes a light pulse to travel along the rod and back (remembering the light pulse travels at the "speed of light", c):

$$d' = \frac{ct'}{2}.$$

Observers in a space station, frame S, see the spaceship pass by at a speed v. How long do they see the rod to be?

They use the exact same method as the scientists in S': they use the time it takes for the light pulse to travel up and down the rod, but the rod is now moving.

If we call the distance travelled by the light pulse in the first part of its journey is s_1 and in the second part s_2 then each distance can be calculated from our diagram as:

$$s_1 = (d + vt_1) = ct_1$$
$$s_2 = (d - vt_2) = ct_2$$

The total time, as seen in S is:

$$t = t_1 + t_2 = \frac{d}{c - v} + \frac{d}{c + v}$$
$$d = \frac{ct}{2}\left(1 - \frac{v^2}{c^2}\right)$$

We then just need to substitute our time dilation result and solve:

$$t = \gamma t',$$

$$\Rightarrow d = \frac{d'}{\gamma}$$

The rod is measured to be smaller in the direction of travel. This is not an optical illusion: the rod is actually shorter when looked at from inertial frames moving relative to S' than when looked at by the observers in S'.

http://www.physicsclassroom.com/mmedia/specrel/lc.cfm

Similarly, the entire spaceship defining S' looks shorter to the observers on the space station. But to the observers on the spaceship, the space station is moving and it is the space station which is contracted. This works because neither is accelerating: both S and S' are inertial frames of reference. If the spaceship was accelerating then the situation would be much more complicated, and we would have to be extremely careful with how we applied relativity, since special relativity only holds for inertial frames. (For those interested: this is achieved by defining "Instantaneous Rest Frames", in which the spaceship is stationary for an infinitesimal time. This treatment is taught in the 3rd year of the Cambridge physics course, so should not concern you yet!)

Worked Example:

A train is 120m long and travelling at speed v. The train is approaching a station, where it won't stop, which has a platform 100m long. How fast does the train need to be going for it to be the same length as the platform, from the inertial frame of the platform?

We call the platform frame S, and the train frame S'. From our length contraction derivation we know that the observer on the platform sees the train contracted therefore $l' > l$ which gives $\gamma l = l'$, so:

$$\frac{l'}{l} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\frac{v}{c} = \sqrt{1 - \frac{l^2}{l'^2}}$$

So substituting the values:

$$\frac{v}{c} = \sqrt{1 - \frac{10{,}000}{14{,}400}} = 0.55c.$$